double angle

[SQA] 1. Solve the equation $3\cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \le x \le 360$. **5**

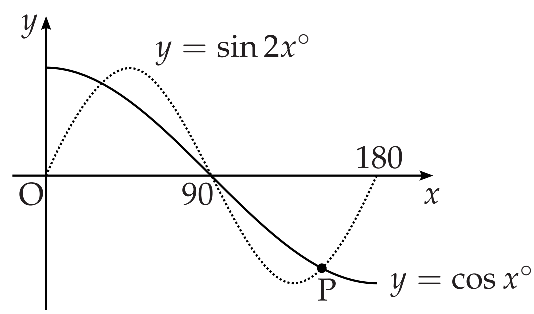
Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	A/B	CR	T10	$60, 131{\cdot}8, 228{\cdot}2, 300$	2000 P2 Q5

•[1] ss: know to use $\cos 2x = 2\cos^2 x - 1$	•[1] $3(2\cos^2 x^\circ - 1)$
•[2] pd: process	•[2] $6\cos^2 x^\circ + \cos x^\circ - 2 = 0$
•[3] ss: know to/and factorise quadratic	•[3] $(2\cos x^\circ - 1)(3\cos x^\circ + 2)$
•[4] pd: process	•[4] $\cos x^\circ = \frac{1}{2}$, $x = 60, 30$
•[5] pd: process	•[5] $\cos x^\circ = -\frac{2}{3}$, $x = 132, 228$

[SQA] 2. (*a*) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \le x \le 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	4	C	NC	T10	$30, 90, 150$	2001 P1 Q5
(*b*)	1	C	NC	T3	$(150, -\frac{\sqrt{3}}{2})$	

•[1] ss: use double angle formula	•[1] $2\sin x^\circ \cos x^\circ$
•[2] pd: factorise	•[2] $\cos x^\circ(2\sin x^\circ - 1)$
•[3] pd: process	•[3] $\cos x^\circ = 0$, $\sin x^\circ = \frac{1}{2}$
•[4] pd: process	•[4] $90, 30, 150$
•[5] ic: interpret graph	**or**
	•[3] $\sin x^\circ = \frac{1}{2}$ and $x = 30, 150$
	•[4] $\cos x^\circ = 0$ and $x = 90$
	•[5] $\left(150, -\frac{\sqrt{3}}{2}\right)$

[SQA] 3. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

(a) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

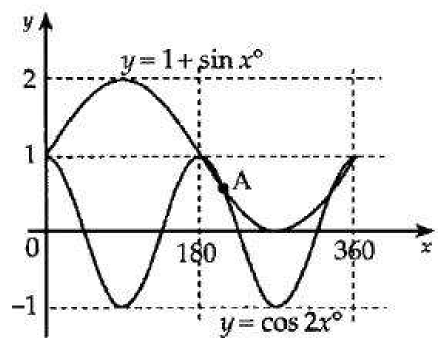
(b) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	2	C	CN	A4	(i) $\sin(2x^\circ)$, (ii) $2\sin(x^\circ)$	2002 P1 Q3
(b)	5	C	CN	T10	$0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$	

•[1] ic: interpret $f(g(x))$ •[2] ic: interpret $g(f(x))$ •[3] ss: equate for intersection •[4] ss: substitute for $\sin 2x$ •[5] pd: extract a common factor •[6] pd: solve a 'common factor' equation •[7] pd: solve a 'linear' equation	•[1] $\sin(2x^\circ)$ •[2] $2\sin(x^\circ)$ •[3] $2\sin(2x^\circ) = 2\sin(x^\circ)$ •[4] appearance of $2\sin(x^\circ)\cos(x^\circ)$ •[5] $2\sin(x^\circ)\,(2\cos(x^\circ) - 1)$ •[6] $\sin(x^\circ) = 0$ **and** $0, 180, 360$ •[7] $\cos(x^\circ) = \frac{1}{2}$ **and** $60, 300$ **or** •[6] $\sin(x^\circ) = 0$ **and** $\cos(x^\circ) = \frac{1}{2}$ •[7] $0, 60, 180, 300, 360$

[SQA] 4. The diagram shows two curves with equations $y = \cos 2x^\circ$ and $y = 1 + \sin x^\circ$ where $0 \leq x \leq 360$.

Find the x-coordinate of the point of intersection at A. **4**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	1	C	NC	T10		1991 P1 Q20
	3	A/B	NC	T10		

•[1] $\cos 2x^\circ = 1 + \sin x^\circ$

•[2] $2\sin^2 x^\circ + \sin x^\circ = 0$

•[3] $\sin x^\circ = 0$ *or* $-\frac{1}{2}$

•[4] $x = 210$

[SQA] 5. Solve the equation $\cos 2x^\circ + 5\cos x^\circ - 2 = 0$, $0 \leq x < 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	1	C	CR	T10		1994 P1 Q15
	4	A/B	CR	T10		

- •1 Replacing $\cos 2x$ by $2\cos^2 x - 1$
- •2 $2\cos^2 x + 5\cos x - 3 = 0$
- •3 $(2\cos x - 1)(\cos x + 3) = 0$
- •4 60°
- •5 300°
  and
  no extraneous solutions
  and
  no solution for $\cos x = -3$ indicated.
  [If a reason is given, it must be valid].

[SQA] 6. Solve the equation $\cos 2x^\circ + \cos x^\circ = 0$, $0 \leq x < 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	A/B	CR	T10		1995 P1 Q15

- •1 substitute $2\cos^2 x^\circ - 1$ for $\cos 2x^\circ$
- •2 $(2\cos x^\circ - 1)(\cos x^\circ + 1) = 0$
- •3 $\cos x^\circ = \frac{1}{2}$, $\cos x^\circ = -1$
- •4 $x = 60, 300$
- •5 $x = 180$

[SQA] 7. Solve the equation $\sin 2x^\circ + \sin x^\circ = 0$, $0 \leq x < 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	C	NC	T10		1996 P1 Q10

- •1 $2\sin x\cos x + \sin x = 0$
- •2 $\sin x(2\cos x + 1) = 0$
- •3 $\sin x = 0$, $\cos x = -\frac{1}{2}$
- •4 1st: $x = 0, 180$
- •5 2nd: $x = 120, 240$

[SQA] 8.

(*a*) Show that $2\cos 2x^\circ - \cos^2 x^\circ = 1 - 3\sin^2 x^\circ$. **2**

(*b*) Hence solve the equation $2\cos 2x^\circ - \cos^2 x^\circ = 2\sin x^\circ$ in the interval $0 \leq x < 360$. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	1	C	CR	T8		1997 P1 Q18
(*a*)	1	A/B	CR	T8		
(*b*)	1	C	CR	T10		
(*b*)	3	A/B	CR	T10		

•1 substitute $1-2\sin^2 x^\circ$ for $\cos 2x^\circ$

•2 substitute $1-\sin^2 x^\circ$ for $\cos^2 x^\circ$

•3 $3\sin^2 x^\circ + 2\sin x^\circ - 1 = 0$

•4 $(3\sin x^\circ - 1)(\sin x^\circ + 1) = 0$

•5 $\sin x^\circ = \frac{1}{3}, -1$

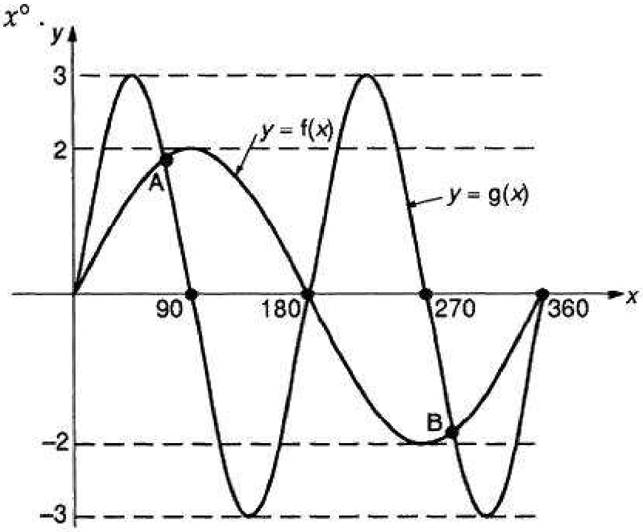
•6 $19\cdot5,\ 160\cdot5,\ 270$

[SQA] 9. (*a*) Solve the equation $3\sin 2x° = 2\sin x°$ for $0 \le x \le 360$ **(4)**

(*b*) The diagram below shows parts of the graphs of sine functions f and g.
State expressions for $f(x)$ and $g(x)$. **(1)**

(*c*) Use your answers to part (*a*) to find the co-ordinates of A and B. **(2)**

(*d*) Hence state the values of x in the interval $0 \le x \le 360$ for which $3\sin 2x° < 2\sin x°$. **(3)**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	4	C	CR	T10		1992 P2 Q7
(*b*)	1	C	CR	A7		
(*c*)	2	C	CR	A6		
(*d*)	2	C	CR	T2		
(*d*)	1	A/B	CR	T2		

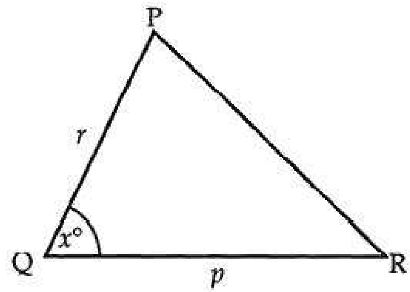
(*a*)
- •[1] strategy: ie $\sin 2x = 2\sin x\cos x$
- •[2] $\sin x = 0$ AND $\cos x = \frac{1}{3}$
- •[3] 0, 180 AND 360
- •[4] 70.5 AND 289.5 AND no other angles

(*b*)
- •[5] $f(x) = 2\sin x°$, $g(x) = 3\sin 2x°$

(*c*)
- •[6] $x = 70.5$ AND 289.5
- •[7] $y = 1.89$ AND -1.89

(*d*)
- •[8] 70.5 AND 180
- •[9] 289.5 AND 360
- •[10] use inequality signs logically to connect the points of intersection (ie not for $180 < x < 70.5$)

[SQA] 10. The diagram shows an isosceles triangle PQR in which PR = QR and angle PQR = $x°$.



(*a*) Show that $\frac{\sin x°}{p} = \frac{\sin 2x°}{r}$. **(3)**

(*b*) (i) State the value of $x°$ when $p = r$.

(ii) Using the fact that $p = r$, solve the equation in (*a*) above, to justify your stated value of $x°$. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	CN	CGD		1991 P2 Q3
(*a*)	1	A/B	CN	CGD		
(*b*)	5	C	CN	T10, T11		

(*a*)
- •[1] $(180-2x)°$
- •[2] $\frac{\sin x°}{p} = \frac{\sin(180-2x)°}{r}$
- •[3] $\sin(180-2x)° = \sin 2x°$ **stated explicitly**

(*b*)
- •[4] $60°$
- •[5] $\sin x° = \sin 2x°$
- •[6] $\sin x°(2\cos x° - 1) = 0$
- •[7] $\sin x° = 0$ *and* $\cos x° = \frac{1}{2}$
- •[8] $x = 60$ is only answer **stated explicitly**

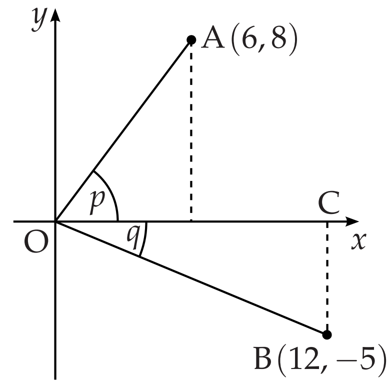
11. Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \leq x < 2\pi$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	B	CN	T10, T7	$x = 2{\cdot}419, 3{\cdot}864$	2010 P2 Q4

•[1] ss: know to use double angle formula	•[1] $2 \times (2\cos^2 x - 1)\cdots$
•[2] ic: express as quadratic in $\cos x$	•[2] $4\cos^2 x - 5\cos x - 6 = 0$
•[3] ss: start to solve	•[3] $(4\cos x + 3)(\cos x - 2) = 0$
•[4] pd: reduce to equations in $\cos x$ only	•[4] $\cos x = -\frac{3}{4}$ and $\cos x = 2$
•[5] pd: complete solutions to include only one where $\cos x = k$ with $\lvert k\rvert > 1$	•[5] 2·419, 3·864 and no solution.

[SQA] 12. On the coordinate diagram shown, A is the point $(6, 8)$ and B is the point $(12, -5)$. Angle AOC $= p$ and angle COB $= q$.

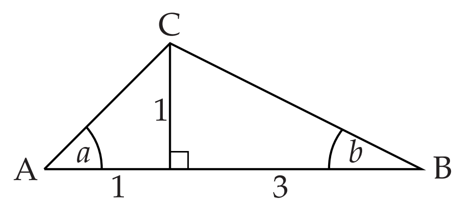
Find the exact value of $\sin(p + q)$. **4**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T9	$\frac{63}{65}$	2000 P1 Q1

•1 ss: know to use trig expansion •2 pd: process missing sides •3 ic: interpret data •4 pd: process	•1 $\sin p \cos q + \cos p \sin q$ •2 10 **and** 13 •3 $\frac{8}{10} \cdot \frac{12}{13} + \frac{6}{10} \cdot \frac{5}{13}$ •4 $\frac{126}{130}$

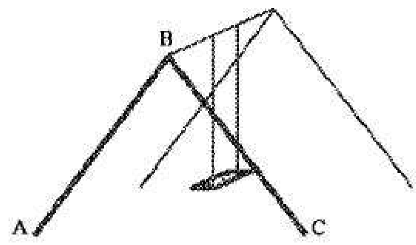
[SQA] 13. In triangle ABC, show that the exact value of $\sin(a + b)$ is $\dfrac{2}{\sqrt{5}}$. **4**

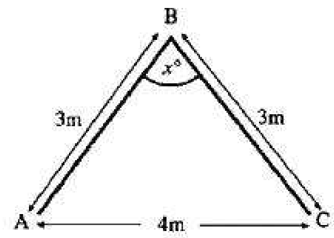


Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T9	proof	2002 P1 Q5

•1 pd: process the missing sides •2 ss: expand •3 pd: substitute •4 pc: process and complete proof	•1 $AC = \sqrt{2}$ **and** $BC = \sqrt{10}$ *stated or implied by* •3 •2 $\sin(a + b) = \sin a \cos b + \cos a \sin b$ •3 $\frac{1}{\sqrt{2}} \cdot \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{10}}$ •4 $\frac{4}{\sqrt{20}} = \ldots = \frac{2}{\sqrt{5}}$

[SQA] 14. The framework of a child's swing has dimensions as shown in the diagram on the right.
Find the exact value of $\sin x°$. **5**

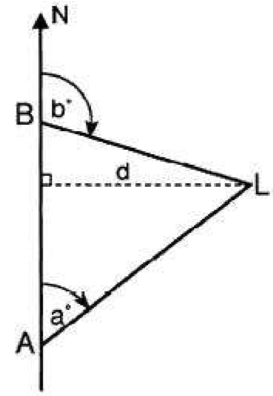




Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	1	C	NC	T9		1996 P1 Q18
	4	A/B	NC	T9		

•¹	sketch with $\frac{x}{2}$ marked in r/a Δ	**OR**	•¹	know to use cosine rule
•²	height of triangle $= \sqrt{5}$		•²	$\cos x = \frac{3^2+3^2-4^2}{2.3.3}$
•³	$\sin x = 2\sin\frac{1}{2}x\cos\frac{1}{2}x$		•³	$\frac{1}{9}$
•⁴	$\sin\frac{x}{2} = \frac{2}{3}$ *and* $\cos\frac{1}{2}x = \frac{\sqrt{5}}{3}$		•⁴	draw r/a Δ *or* use $\cos^2 x + \sin^2 x = 1$
•⁵	$\sin x = \frac{4\sqrt{5}}{9}$		•⁵	$\sin x = \frac{\sqrt{80}}{9}$

[SQA] 15. A ship is sailing due north at a constant speed. When at position A, lighthouse L is observed on a bearing of $a°$. One hour later, when the ship is at position B, the lighthouse is on a bearing of $b°$. The shortest distance between the ship and the lighthouse during this hour was d miles.



(a) Prove that $AB = \dfrac{d}{\tan a°} - \dfrac{d}{\tan b°}$. **(2)**

(b) Hence prove that $AB = \dfrac{d\sin(b-a)°}{\sin a° \sin b°}$. **(3)**

(c) Calculate the shortest distance from the ship to the lighthouse when the bearings $a°$ and $b°$ are 060° and 135° respectively and the constant speed of the ship is 14 miles per hour. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	1	C	CR	CGD		1992 P2 Q8
(a)	1	A/B	CR	CGD		
(b)	3	A/B	CR	T9		
(c)	3	C	CR	CGD		

(a) •[1] $CA = \frac{d}{\tan a}$

•[2] $CB = \frac{d}{\tan(180-b)}$

(b) •[3] $AB = \frac{d}{\frac{\sin a}{\cos a}} - \frac{d}{\frac{\sin b}{\cos b}}$

•[4] $\frac{d\cos a}{\sin a} - \frac{d\cos b}{\sin b}$

•[5] $\frac{d\sin b\cos a - d\cos b\sin a}{\sin a\sin b}$

(c) •[6] AB = 14

•[7] 1.577 or 0.634

(comes from $AB = 1.577d$ or $d = 0.634\ AB$)

•[8] 8.9 miles

[SQA] 16.

(*a*) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. **3**

(*b*) Show that $\sin(A + B) + \sin(A - B) = 2\sin A\cos B$. **2**

(*c*) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

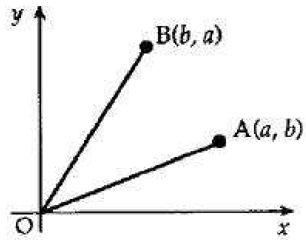
(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	3	C	NC	T8, T3	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	2009 P1 Q24
(*b*)	2	C	CN	T8	proof	
(*c*)	3	B	NC	T11	$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$	
(*c*)	1	C	NC	T11	$\frac{\sqrt{6}}{2}$ or $\sqrt{\frac{3}{2}}$	

•1 ss: expand compound angle	•1 $\sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$
•2 ic: substitute exact values	•2 $\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$
•3 pd: process to a single fraction	•3 $\frac{\sqrt{3}+1}{2\sqrt{2}}$ or equivalent
•4 ic: start proof	•4 $\sin A\cos B + \cos A\sin B + \cdots$
•5 ic: complete proof	•5 $\cdots + \sin A\cos B - \cos A\sin B$ and complete
•6 ss: identify steps	•6 $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$
•7 ic: start process (identify 'A' & 'B')	•7 $2 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$
•8 ic: substitute	•8 $\frac{\sqrt{6}}{2}$ or $\sqrt{\frac{3}{2}}$
•9 pd: process	

[SQA] 17. In the diagram, A and B have coordinates as shown.

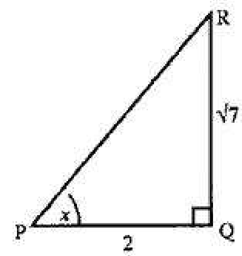
Express $\sin A\hat{O}B$ in terms of a and b. **4**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	A/B	NC	T8		1989 P1 Q11

- •1 $OA = OB = \sqrt{a^2 + b^2}$
- •2 $\sin A\hat{O}B = \sin\left(B\hat{O}x - A\hat{O}x\right) = \sin B\hat{O}x \cos A\hat{O}x - \cos B\hat{O}x \sin A\hat{O}x$
- •3 $\frac{a}{\sqrt{a^2+b^2}} \cdot \frac{a}{\sqrt{a^2+b^2}} - \frac{b}{\sqrt{a^2+b^2}} \cdot \frac{b}{\sqrt{a^2+b^2}}$
- •4 $\frac{a^2-b^2}{a^2+b^2}$

[SQA] 18. Using triangle PQR, as shown, find the exact value of $\cos 2x$. **3**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	3	C	NC	T8		1999 P1 Q12

- •1 $\cos x = \frac{2}{\sqrt{11}}$ *or* $\sin x = \frac{\sqrt{7}}{\sqrt{11}}$
- •2 $\cos 2x = 2 \times \left(\frac{2}{\sqrt{11}}\right)^2 - 1$
- •3 $-\frac{3}{11}$

[SQA] 19. Given that $\cos D = \frac{2}{\sqrt{5}}$ and $0 < D < \frac{\pi}{2}$, find the exact values of $\sin D$ and $\cos 2D$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T8		1990 P1 Q9

- •1 *strat for exact value: e.g.* $\sin^2 D = 1 - \cos^2 D$
- •2 $\sin D = \frac{1}{\sqrt{5}}$
- •3 $\cos 2D = \frac{3}{5}$

[SQA] 20. Given that $\sin A = \frac{3}{4}$, where $0 < A < \frac{\pi}{2}$, find the exact value of $\sin 2A$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	3	C	NC	T8		1991 P1 Q12

•[1] *strat for* cos: *eg* $\cos^2 = 1 - \sin^2$

•[2] $\cos A = \frac{\sqrt{7}}{4}$

•[3] $\sin 2A = \frac{3\sqrt{7}}{8}$

[SQA] 21. For acute angles P and Q, $\sin P = \frac{12}{13}$ and $\sin Q = \frac{3}{5}$.

Show that the exact value of $\sin(P + Q)$ is $\frac{63}{65}$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	3	C	NC	T8		1993 P1 Q6

•[1] $\cos P = \frac{5}{13}$

•[2] $\cos Q = \frac{4}{5}$

•[3] $\frac{12}{13} \times \frac{4}{5} + \frac{5}{13} \times \frac{3}{5}$

[SQA] 22. Find the exact value of $\sin\theta^\circ + \sin(\theta^\circ + 120^\circ) + \cos(\theta^\circ + 150^\circ)$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	3	C	NC	T8		1994 P1 Q6

•[1] $\sin\theta\cos 120 + \cos\theta\sin 120$ **and** $\cos\theta\cos 150 - \sin\theta\sin 150$

•[2] correct use of exact values

•[3] simplification to zero

[SQA] 23. If $\cos\theta = \frac{4}{5}$, $0 \le \theta < \frac{\pi}{2}$, find the exact value of

(a) $\sin 2\theta$ **2**

(b) $\sin 4\theta$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	2	C	NC	T8		1994 P1 Q13
(b)	3	A/B	NC	T8		

•1 $\sin\theta = \frac{3}{5}$

•2 $\frac{24}{25}$

•3 $2\sin 2\theta\cos 2\theta$

•4 $\cos 2\theta = \frac{7}{25}$

•5 $\frac{336}{625}$

[SQA] 24. Given that $\tan\alpha = \frac{\sqrt{11}}{3}$, $0 < \alpha < \frac{\pi}{2}$, find the exact value of $\sin 2\alpha$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	3	C	NC	T8		1995 P1 Q12

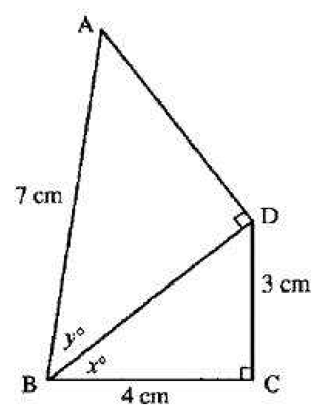
•1 "third side" $= \sqrt{20}$

•2 $\sin\alpha = \frac{\sqrt{11}}{\sqrt{20}}$ *or* $\cos\alpha = \frac{3}{\sqrt{20}}$

•3 $2 \times \frac{\sqrt{11}}{\sqrt{20}} \times \frac{3}{\sqrt{20}}$

[SQA] 25. The diagram shows two right-angled triangles ABD and BCD with AB = 7cm, BC = 4cm and CD = 3cm. Angle DBC = $x°$ and angle ABD = $y°$.

Show that the exact value of $\cos(x+y)°$ is $\frac{20-6\sqrt{6}}{35}$. 3



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	3	C	CN	T8		1996 P1 Q15

•1 know to calculate missing sides

•2 BD = 5, AD = $\sqrt{24}$

•3 $\cos x\cos y - \sin x\sin y = \frac{4}{5}.\frac{5}{7} - \frac{3}{5}.\frac{\sqrt{24}}{7}$

[SQA] 26. If x° is an acute angle such that $\tan x^\circ = \frac{4}{3}$, show that the exact value of $\sin(x^\circ + 30^\circ)$ is $\dfrac{4\sqrt{3}+3}{10}$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	3	C	NC	T8		1997 P1 Q7

•1 $\sin x^\circ \cos 30^\circ + \cos x^\circ \sin 30^\circ$

•2 $\sin x^\circ = \frac{4}{5}$ & $\cos x^\circ = \frac{3}{5}$

•3 $\frac{4}{5}.\frac{\sqrt{3}}{2} + \frac{3}{5}.\frac{1}{2}$ and completes proof

[SQA] 27. A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$.

Find the exact value of

(*a*) $\sin 2A$ **2**

(*b*) $\cos 2A$ **1**

(*c*) $\sin(2A + B)$. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	NC	T8		1998 P1 Q7
(*b*)	1	C	NC	T8		
(*c*)	2	C	NC	T8		

•1 $\sin A = \frac{3}{5}$ and $\cos A = \frac{4}{5}$

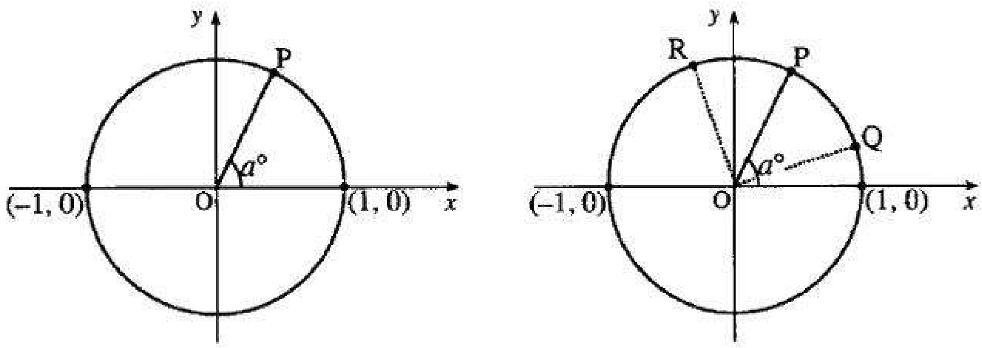
•2 $\sin 2A = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$ (*accept* 0.96)

•3 $\cos 2A =$ *e.g.* $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$ (*accept* 0.28)

•4 $\sin 2A \cos B + \cos 2A \sin B$

•5 $\sin B = \frac{5}{13}$ *and* $\cos B = \frac{12}{13}$ *and* $\frac{323}{325}$

[SQA] 28. The diagram shows a circle of radius 1 unit and centre the origin. The radius OP makes an angle $a°$ with the positive direction of the x-axis.



(*a*)	Show that P is the point $(\cos a°, \sin a°)$.	1
(*b*)	If $P\hat{O}Q = 45°$, deduce the coordinates of Q in terms of a.	1
(*c*)	If $P\hat{O}R = 45°$, deduce the coordinates of R in terms of a.	1
(*d*)	Hence find an expression for the gradient of QR in its simplest form.	4
(*e*)	Show that the tangent to the circle at P is parallel to QR.	2

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a-c*)	3	C	NC	CGD		1999 P2 Q8
(*d*)	4	A/B	NC	T8, G2		
(*e*)	2	A/B	NC	G2, G5		

(*a*) •1 proof e.g. showing rt - angled triangle with "1" and a°

(*b*) •2 Q is $(\cos(a-45)°, \sin(a-45)°)$

(*c*) •3 R is $(\cos(a+45)°, \sin(a+45)°)$

(*d*) •4 $\frac{\sin(a+45)-\sin(a-45)}{\cos(a+45)-\cos(a-45)}$

•5 $\frac{\sin a\cos 45+\cos a\sin 45-\sin a\cos 45+\cos a\sin 45}{\cos a\cos 45-\sin a\sin 45-\cos a\cos 45-\sin a\sin 45}$

•6 $\frac{2\cos a\sin 45}{-2\sin a\sin 45}$

•7 $-\frac{1}{\tan a}$

(*e*) •8 $m_{OP} = \frac{\sin a}{\cos a} = \tan a$

•9 $m_{tgt\ at\ P} = -\frac{1}{\tan a}$

29. (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of $\sin a$. **4**

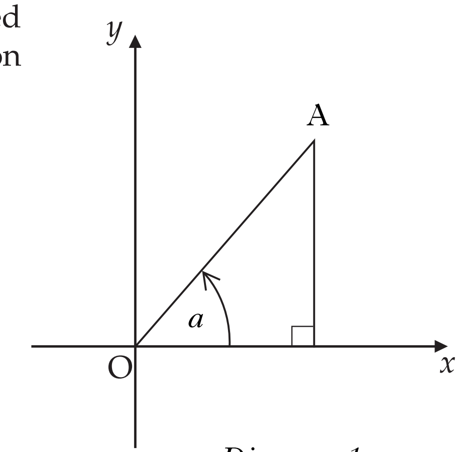


Diagram 1

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$. **4**

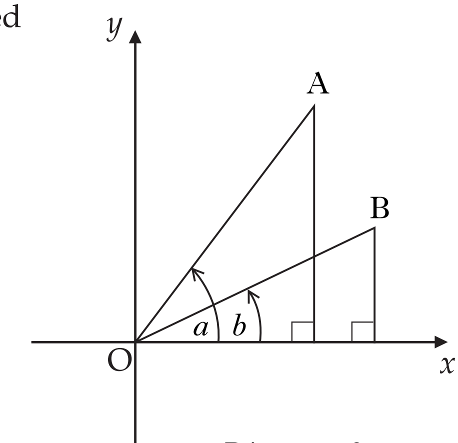


Diagram 2

(*c*) (i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	4	C	CN	G2, T5	proof, $\frac{3}{\sqrt{13}}$	2010 P1 Q23
(*b*)	4	C	CN	G2, T5	$\sin b = \frac{3}{5}, \cos b = \frac{4}{5}$	
(*c*)	4	B	CN	T8, T1	$\frac{6}{5\sqrt{13}}, -\frac{6}{5\sqrt{13}}$	

•1 ss: write in slope/intercept form	•1 $y = \frac{3}{2}x$
•2 ic: connect gradient and $\tan a$	•2 $m = \frac{3}{2}$ and $\tan a = \frac{3}{2}$
•3 pd: calculate hypotenuse	•3 $\sqrt{13}$
•4 ic: state value of sine ratio	•4 $\frac{3}{\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$
•5 ss: determine $\tan b$	•5 $\tan b = \frac{3}{4}$
•6 ss: know to complete triangle	•6 right-angled triangle with 3, 4
•7 pd: determine hypotenuse	•7 5
•8 ic: state values of sine and cosine ratios	•8 $\sin b = \frac{3}{5}$ and $\cos b = \frac{4}{5}$
•9 ss: know to use addition formula	•9 $\sin a \cos b - \cos a \sin b$
•10 ic: substitute into expansion	•10 $\frac{3}{\sqrt{13}} \times \frac{4}{5} - \frac{2}{\sqrt{13}} \times \frac{3}{5}$
•11 pd: evaluate sine of compound	•11 $\frac{6}{5\sqrt{13}}$
	•12 $-\frac{6}{\ldots}$

[SQA] 30.

(*a*) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that, for $\cos\theta$, it has equal roots. **3**

(*b*) Show that there are no real roots for θ. **1**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	1	C	CN	T8, A17		1998 P1 Q18
(*a*)	2	A/B	CN	T8, A17		
(*b*)	1	A/B	CN	A1		

•¹ $2\cos^2\theta - 1 + 8\cos\theta + 9$	•⁴ $\cos\theta = -2$ has no solution
•² $2(\cos\theta + 2)^2 = 0$	
or "$b^2 - 4ac$" $= 16 - 4\times1\times4$	
•³ $\cos\theta = -2$ twice or "$b^2 - 4ac$" $= 0$	

[SQA] 31. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \le x \le 2\pi$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	NC	A4	(i) *sin*$(x+\frac{\pi}{4})$, (ii) *cos*$(x+\frac{\pi}{4})$	2001 P1 Q7
(*b*)	5	C	NC	T8, T7	(i) proof, (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$	

•¹ ic: interpret composite functions	•¹ $\sin(x+\frac{\pi}{4})$
•² ic: interpret composite functions	•² $\cos(x+\frac{\pi}{4})$
•³ ss: expand $\sin(x+\frac{\pi}{4})$	•³ $\sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4}$ **and complete**
•⁴ ic: interpret	•⁴ $g(h(x)) = \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$
•⁵ ic: substitute	•⁵ $(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x) - (\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x)$
•⁶ pd: start solving process	•⁶ $\frac{2}{\sqrt{2}}\sin x$
•⁷ pd: process	•⁷ $x = \frac{\pi}{4}, \frac{3\pi}{4}$ *accept only radians*

[END OF QUESTIONS]